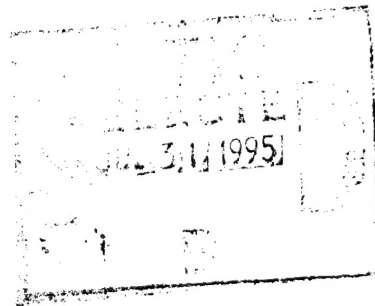


The US Army's Center for Strategy and Force Evaluation

RESEARCH PAPER
CAA-RP-95-4

INCORPORATION OF LEARNING CURVE COSTS IN ACQUISITION STRATEGY OPTIMIZATION

JULY 1995



PREPARED BY
VALUE ADDED ANALYSIS DIVISION

US ARMY CONCEPTS ANALYSIS AGENCY
8120 WOODMONT AVENUE
BETHESDA, MARYLAND 20814-2797

19950728 012



DISCLAIMER

The findings of this report are not to be construed as an official Department of the Army position, policy, or decision unless so designated by other official documentation. Comments or suggestions should be addressed to:

Director
US Army Concepts Analysis Agency
ATTN: CSCA-RSV
8120 Woodmont Avenue
Bethesda, MD 20814-2797

REPORT DOCUMENTATION PAGE			<i>Form Approved</i> <i>OPM NO. 0704-0188</i>	
Public reporting burden for this collection information is estimated to 1 hour per response, including the time for reviewing instructions, searching existing data sources gathering and maintaining the data needed, and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information. Including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of information and Regulatory Affairs, Office of Management and Budget, Washington, DC 20503.				
1. AGENCY USE ONLY (Leave Blank)		2. REPORT DATE July 1995		3. REPORT TYPE AND DATES COVERED Final, Jul 94 - Jul 95
4. TITLE AND SUBTITLE Incorporation of Learning Curve Costs in Acquisition Strategy Optimization			5. FUNDING NUMBER	
6. AUTHOR(S) LTC Andrew G. Loerch				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) US Army Concepts Analysis Agency 8120 Woodmont Avenue Bethesda, MD 20814-2797			8. PERFORMING ORGANIZATION REPORT NUMBER CAA-RP-95-4	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) US Army Concepts Analysis Agency			10. SPONSORING/ MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited			12b. DISTRIBUTION CODE A	
13. ABSTRACT (Maximum 200 words) Each year, the United States Army procures billions of dollars worth of weapons and equipment. The process of deciding what to buy, in what quantities, and when is extremely complex, requiring extensive analysis. Two techniques used in this analysis are mathematical programming and cost estimation. Although they are related through constraints on available procurement funds, the use of nonlinear cost learning curves, which better represent system costs as a function of quantity produced, have not been incorporated into the mathematical programming formulations that compute the quantities of items to be procured. As a result, the solutions obtained could be either suboptimal, or even infeasible with respect to budgetary limitations. In this paper, we present a mixed integer linear programming formulation that uses a piecewise linear approximation of the learning curve costs for a more accurate portrayal of budgetary constraints. In addition, implementation issues are discussed, and performance results are given.				
14. SUBJECT TERMS Learning curves, mathematical programming, capital budgeting			15. NUMBER OF PAGES 26	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL	

Incorporation of Learning Curve Costs in Acquisition Strategy Optimization

ABSTRACT

Each year, the United States Army procures billions of dollars worth of weapons and equipment. The process of deciding what to buy, in what quantities, and when is extremely complex, requiring extensive analysis. Two techniques used in this analysis are mathematical programming and cost estimation. Although they are related through constraints on available procurement funds, the use of nonlinear cost learning curves, which better represent system costs as a function of quantity produced, have not been incorporated into the mathematical programming formulations that compute the quantities of items to be procured. As a result, the solutions obtained could be either suboptimal, or even infeasible with respect to budgetary limitations. In this paper, we present a mixed integer linear programming formulation that uses a piecewise linear approximation of the learning curve costs for a more accurate portrayal of budgetary constraints. In addition, implementation issues are discussed, and performance results are given.

Accession For	
WHIS OGA&I	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

Opinions, interpretations, conclusions, and recommendations are those of the author, and are not necessarily endorsed by the US Army.

The research described herein was sponsored by the US Army Concepts Analysis Agency, and as such is not subject to copyright laws.

Incorporation of Learning Curve Costs in Acquisition Strategy Optimization

1. INTRODUCTION

Each year, the United States Army procures billions of dollars worth of weapons and equipment so that it can accomplish its worldwide mission of deterrence. The process of deciding what to buy to best respond to the threat is extremely complex, thus requiring extensive analysis. Typically cost benefit analysis is used as a means of determining relative return.

Two common techniques used in this analysis are mathematical programming and cost estimation. Although they are related through constraints on available procurement funds, the use of nonlinear cost learning curves, which better represent system costs as a function of quantity produced, have not been incorporated into the mathematical programming formulations that compute the quantities of items to be procured. As a result, the solutions obtained could be either suboptimal, or even infeasible with respect to budgetary limitations.

Mathematical programming has been frequently used to determine an optimal, in some sense, funding and acquisition stream for procurement of Army equipment. Several of these efforts are described below.

In 1984, the Resource Constrained Procurement Objectives for Munitions (RECPOM-85) Study was performed by the US Army Concepts Analysis Agency (CAA). See Levy [10]. Its purpose was to develop an optimization model that could be used to calculate the best mix of ammunition to procure such that the effectiveness of the force would be maximized. A stated limitation of the RECPOM optimization model was that it could not consider the costs of the ammunition as a function of number of items produced. As an alternative, an average unit cost of the items was used that did not vary with quantity. This limitation was never resolved and greatly limited the usefulness of the methodology.

A more general methodology for optimizing acquisition strategy was developed as part of the Army Aviation Modernization Tradeoff Requirements (AAMTOR) Study, a joint effort between CAA and the Naval Postgraduate School. See Brown, et. al. [2]. The optimization model, known as Phoenix, is a large-scale, mixed integer program whose objective is to find the minimum cost set of equipment quantities, as well as finding the best, with respect to cost, timing of the production periods for these items.

A simplification of the Phoenix Model, called Force Modernization Analyzer (FOMOA), was developed at CAA to perform Phoenix-like analyses when the production campaigns for the systems under consideration are given and fixed. See Coblenz [3]. FOMOA is a relatively fast-running model, and its spreadsheet configuration allows easy data input and output display. This model has been used to produce acquisition strategies for armored systems, wheeled vehicles, and helicopters.

Neither Phoenix nor FOMOA considers learning curve costs of the systems to be procured. Schwabauer and Nedimala [12] suggest the following techniques.

In order to address this problem, production quantities are assumed, a priori, likely to be near optimal. Costs are computed that are associated with these quantities which are then used in the budgetary constraints. When the optimization is run, new production quantities are computed which may or may not resemble the a priori quantities. Attempts have been made to iteratively produce costs and quantities in hopes of convergence. However, no such convergence has been shown to be guaranteed.

Phoenix and FOMOA have been used to produce acquisition strategies for various systems such as helicopters or trucks. The need arose to provide optimized acquisition strategies across system types. The Value Added Analysis (VAA) methodology was developed by CAA to provide these strategies, as well as other analysis to support the decision-making necessary to build the Army budget. See US Army [14] for a description of this process. In this analysis, unlike those previously discussed, all of the high priced developmental weapon systems and other equipment are considered and compete for shrinking budgetary resources. As such, the need to accurately represent system procurement costs is critical to the success of the analysis. Also, use is made of nonlinear cost-quantity relationships in the "by hand" calculations made in the programming and budgeting process by staff officers supporting the decision makers. In order to be consistent with the rest of the budgeting work, these relationships must be included in the Value Added analysis as well. See Koury and Loerch [9] for a detailed description of the VAA study.

In the remainder of this paper, we present a mixed integer programming formulation that uses a piecewise linear approximation of the learning curve costs for a better portrayal of budgetary constraints. The implementation of this formulation represents the optimization module of the VAA methodology. This optimization model was used extensively to perform the Army Program support analyses described above.

2. LEARNING CURVES

Learning curves are used to mathematically represent the concept that the more items of a particular type a factory produces, the less each item will cost. Also known as "progress curves," "improvement curves," and "experience curves," they were developed for use in the aircraft industry before and during World War II. Since then, the technique has spread to many other industries. The literature is full of publications describing applications, justifications, and forms for learning curves. In the thirties, Wright [15] described the use of learning curves in the aircraft industry. Since then, literally hundreds of articles on learning curves have been published. Dutton, Thomas, and Butler [5] review about 300 such articles.

2.1 Background

The current atmosphere of defense cuts requires that close attention be paid to weapon systems costing. The front page of the Washington Post on November 9, 1991, carried the following headline:

"Cuts in Defense Budget Create New Inefficiencies."

The article by Pearlstein [11] accompanying this headline describes how reducing the total quantities procured of particular weapons increases the unit cost. This effect results in less savings than anticipated from defense cuts. Learning curve effects that influence the costs of weapon system acquisition alternatives must be considered to better understand the impact of particular changes in defense investment.

The Federal government in general, and the Department of Defense in particular has mandated the use of learning curve costing for cost estimation of acquisition systems. See Department of Defense [4]. Each government contractor must submit a form quantitatively describing the anticipated learning behavior of their manufacturing process. These data are used by cost analysts throughout the acquisition process for determining contract prices, budgetary projections, and for performing cost and effectiveness analyses.

Many forms exist to mathematically depict learning effects. Probably the most popular is the so-called "power", or "exponential", form which is represented as follows:

$$C(y) = Ay^{-b},$$

where

y = the cumulative number of items produced,

C(y) = unit cost of the yth item produced,

A = the cost of the first unit produced, and

b = the learning parameter.

Kanton and Zangwill [8] have suggested that this form of the learning curve is deficient in that it cannot remain form-invariant under aggregation of costs over the subcomponents of the item. However, Stump [13] describes and justifies a method to estimate composite learning curves in the power form, overcoming this objection. This estimate seems to do well for computing costs at the system level, rather than the component level, and the system view is regarded as the appropriate one for the Department of the Army program and budget development.

2.2 Learning Curve Models

Although many variants of the learning curve power model exist, see Yelle [16], two are commonly used in these applications. They are the cumulative average theory model and the unit theory model. Both are described below.

In this notation, C(y) for the cumulative average theory model represents the average cost of all units through the yth unit. Therefore the cumulative cost of producing the first y items is written as

$$[C(y)] y = (Ay^{-b})y = Ay^{1-b}.$$

To compute the cost of producing some consecutive subset of the units produced, say from the first unit produced in a lot, y_f, to the last unit produced in the lot, y_l, the following is used:

$$\text{Cumulative lot cost} = C(y_l)y_l - C(y_f-1)(y_f-1) = A[y_l^{1-b} - (y_f-1)^{1-b}].$$

The cumulative average theory is used when the units are produced individually.

When the items are produced in lots, the unit theory variant is used. In this model, $C(y)$ represents the cost of the y^{th} unit produced. So, to compute the cost of producing a lot of items using this model, the item that has the average cost of all the items in the lot must be identified. This algebraic lot midpoint, denoted Q , is well known, and is computed as:

$$Q = \left[\frac{(y_l - y_f + 1)(1 - b)}{(y_l + 0.5)^{1-b} (y_f - 0.5)^{1-b}} \right]^{\frac{1}{b}} \quad (1)$$

Then the cost of a lot is calculated as follows:

$$\text{Cumulative lot cost} = A Q^{-b} (y_l - y_f + 1).$$

Although these models are related, the values of the their respective learning curve parameters, A and b , would not be the same when estimates for both are made for the same system. Choice of models is made based on the characteristics of the manufacturing process, as well as statistical tests of fit.

2.3 Example of Learning Curve Cost Effects

To demonstrate the importance of correctly assessing the costs of major weapon systems as they relate to quantities procured, we consider a representative sample of such systems that are competing for funding by the US Army. Although the systems themselves are not identified, the learning curves associated with their variable production costs, as computed with the unit theory model, are given in Table 1. As this table shows, the costs of the systems vary depending on the learning curve parameters and the quantities of the items procured. Note that constraints on year by year procurement quantities are imposed, restricting them to be no less than the minimum sustaining rate (MSR), below which it is economically infeasible to produce the items, and no greater than the maximum production rate (MPR), above which the existing or planned production facilities cannot produce the items. Over this range, the difference in unit cost varies by between 16 and 44% for this representative group.

System	A	b	MSR QTY	MPR QTY	Unit Cost MSR	Unit Cost MPR	% Diff
1	1.63	0.315	1200	7500	0.11	0.06	44
2	12.8	0.155	72	228	5.14	4.29	16
3	66.5	0.209	72	168	19.54	16.37	16
4	2.04	0.216	240	864	0.44	0.34	24
5	7.75	0.209	12	48	2.75	1.92	30

Table 1. Sample of Learning Curve Systems (costs in \$ Millions)

These effects of learning curves on unit costs are significant, especially when taken in the context of the estimated costs of yearly procurement lots. Figures 1a, 1b, and 1c depict the yearly effect of the difference in lot costs resulting from the learning curve effects estimated for systems 1, 2, and 3 from Table 1, respectively. Plotted in each of these figures is the yearly cost of a production lot whose size is the arithmetic mean of the MSR and MPR computed using the MSR, the MPR, and the average lot size as the basis for calculating the lot cost. This cost of the average lot would likely be used if the learning curve unit costs could not be computed in the mathematical programming model. Note that yearly differences in excess of \$200 million are not uncommon, representing departures from the cost estimates used by staff budget analysts. Failure to account for these differences reduces the usefulness of the acquisition strategy optimizer.

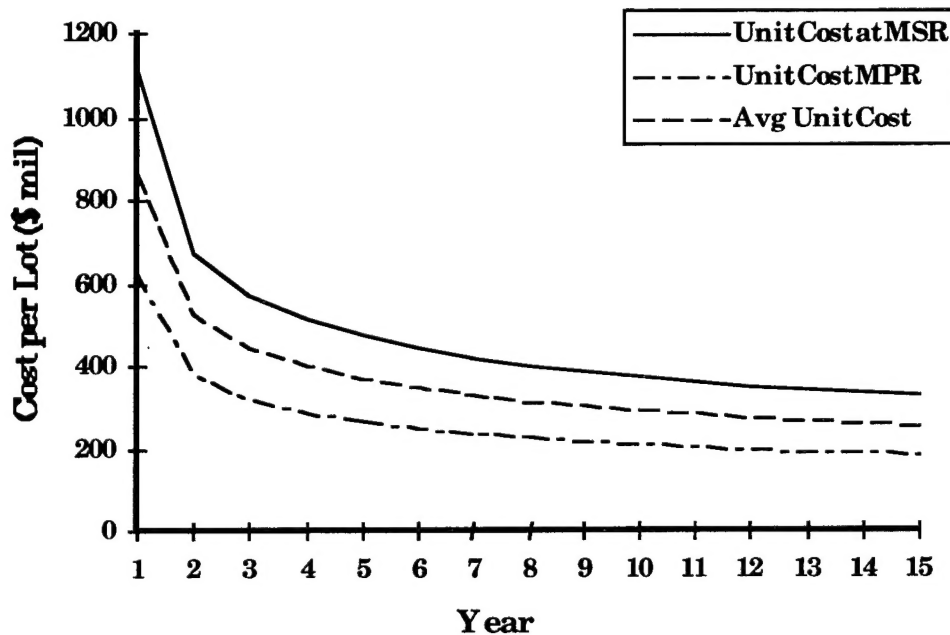


Figure 1a. Yearly Lot Cost Comparison for System 1

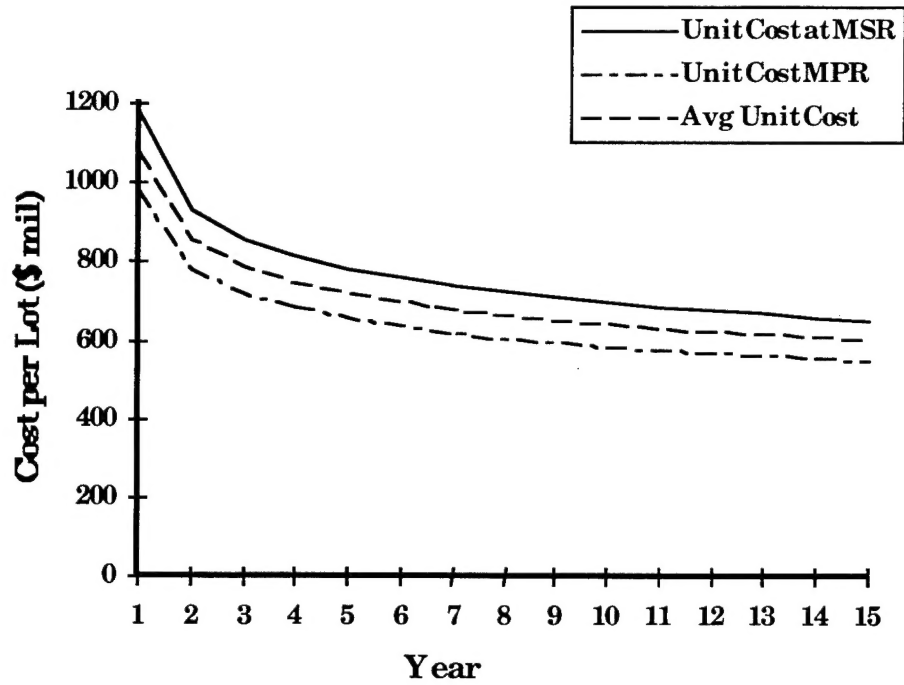


Figure 1b. Yearly Lot Cost Comparison for System 2

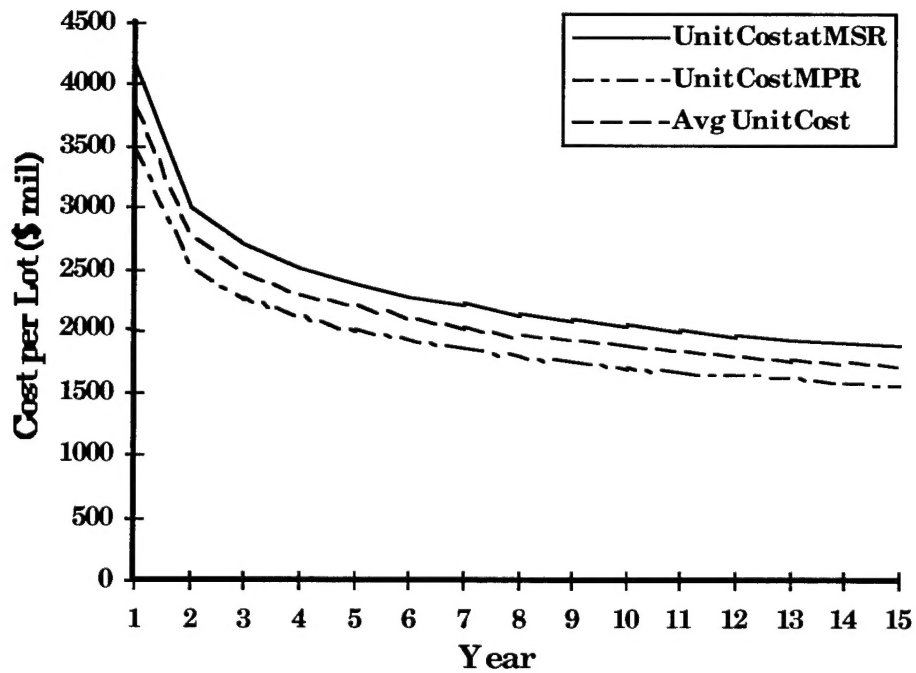


Figure 1c. Yearly Lot Cost Comparison for System 3

3. MATHEMATICAL FORMULATION

The purpose of the VAA optimization model is to produce a "good" and feasible acquisition strategy for the procurement of weapon systems and equipment over a 15-year period. This optimization model utilizes a formulation similar to those of Phoenix and FOMOA and can be represented as follows.

Maximize: Force effectiveness
Subject to: Budget ceiling by year
Force structure requirements
Production limitations

The method developed to incorporate learning curve costing into the optimization formulation is presented in this section.

3.1 Objective Function

The objective of the optimization is to suggest a mix of systems for procurement that will produce a force that is as effective as possible in combat, subject to constraints on budget, force structure, and production capabilities. The contribution of the various candidate systems to the effectiveness of the fighting force with respect to several factors is evaluated and quantified, and these various measures are integrated using a multi-attribute decision-making methodology. The result is a single measure of a system's contribution to the effectiveness of the overall force for each year the system will be in the force. This measure is then used to form the objective function coefficient, v_{ij} , which is the per-item contribution of the system to force effectiveness. Let x_{ij} be defined as the quantity of system i procured in year j , where $j=1, \dots, n$, with n being the number of years in the planning horizon. Considering m systems in the analysis, the objective function is written as

$$\text{Maximize } \sum_{j=1}^n \sum_{i=1}^m v_{ij} x_{ij}.$$

3.2 Constraints on Production Quantities

As discussed above, the production quantities for each system in each year of production are constrained to be between the MSR, denoted $P_{\min_{ij}}$, and the MPR, denoted $P_{\max_{ij}}$. Also, the total quantities produced over the life of the procurement of a system must be sufficient to modernize a specified force. This force is typically identified by the study sponsors, and might be stated in terms of a particular set of units, such as the Contingency Corps; a category of units, such as all light divisions; or some acceptable range of the total quantity might be specified, denoted R_{\min_i} to R_{\max_i} .

The procurement quantities for some systems are restricted in the sense that wide swings in the number produced in successive years are not possible. So constraints are added to preclude differences in production quantities in successive years beyond some given percentage, p_i .

Finally, we want to account for the possibility that a system might not be recommended for procurement, meaning that a feasible solution can have procurement quantities of zero, as well as within the specified range. To deal with this possibility we introduce a binary variable defined as follows:

$$u_i = \begin{cases} 1, & \text{if system } i \text{ is recommended for procurement,} \\ 0, & \text{otherwise.} \end{cases}$$

Then the following constraints are used:

$$P_{\min_{ij}} u_i \leq x_{ij} \leq P_{\max_{ij}} u_i; \quad i = 1, \dots, m; \quad j = 1, \dots, n;$$

$$(1 - p_i)x_{ij-1} \leq x_{ij} \leq (1 + p_i)x_{ij-1}; \quad i = 1, \dots, m; \quad j = 2, \dots, n;$$

$$R_{\min_i} u_i \leq \sum_{j=1}^n x_{ij} \leq R_{\max_i}; \quad i = 1, \dots, m.$$

3.3 Budgetary Constraints

We note at the outset that there is a separate budgetary constraint for each year in the time period, j , of interest for the study. In each year, no more than B_j can be expended for procurement in year j . Funds designated for use in a particular year cannot be carried over into following years. The model will maximize effectiveness which is accumulated in the objective by the procurement of equipment through the expenditure of funds. The funds available in each year are specified in these budgetary constraints, so their accuracy is extremely important to obtaining a valid solution. In this section we develop an approximate method for representing the learning curve costs that will ensure this accuracy.

We introduce the following additional notation.

A_i = first unit cost of system i ,

b_i = cost/quantity slope parameter of system i , and

y_{ij} = cumulative number of system i produced through year j ,

$$= \sum_{k=1}^j x_{ik}.$$

Three components of costs are considered in the budget constraints. These components are fixed costs, variable production costs without learning behavior, and variable production costs with learning behavior. Following is a description of each.

Fixed costs are assessed when a program is implemented. They are not a function of the quantity procured. There are two types of fixed costs that are considered. The first type is research, development, test and evaluation (RDTE)

cost which is incurred prior to production. The second is fixed production cost which is usually assessed prior to and during the first few years of production. Fixed costs are denoted as \tilde{c}_{ij} , representing the sum of the RDTE and fixed production costs of system i in year j , and are assessed only if the system is recommended for procurement. That is if $u_i = 1$, then \tilde{c}_{ij} is assessed by the model. Otherwise no fixed costs are assessed. RDTE expenditures made prior to the 15 year period of analysis are considered sunk.

The costs for some candidate systems either exhibit no learning behavior, or there may be some component of the cost that exhibits no learning behavior. Thus the option for a "non-learning" variable cost is included in the formulation of the budget constraints. This cost component is denoted as \bar{c}_{ij} , the non-learning cost of producing an item of system i in year j .

Finally consider the learning curve costs. Using one of the variants of the power form of the learning curve as described above, we denote learning curve portion of the cost of a lot, which is a function of the lot quantity, x_{ij} , as well as the quantity produced prior to that lot, y_{ij-1} , as $\hat{C}(x_{ij}, y_{ij-1})$, where

$$\hat{C}(x_{ij}, y_{ij-1}) = A_i \left[\frac{(x_{ij})(1-b_i)}{(y_{ij-1} + x_{ij} + 0.5)^{1-b_i} (y_{ij-1} + 0.5)^{1-b_i}} \right]^{\frac{1}{b_i}} x_{ij},$$

when the unit theory is used, and

$$\hat{C}(x_{ij}, y_{ij-1}) = A_i (y_{ij-1} + x_{ij})^{1-b_i} - A_i (y_{ij-1})^{1-b_i},$$

when the cumulative average theory is used.

The budget constraints are then written as:

$$\sum_{i=1}^m [\tilde{c}_{ij}u_i + \bar{c}_{ij}x_{ij} + \hat{C}(x_{ij}, y_{ij-1})] \leq B_j, \quad j = 1, \dots, n. \quad (2)$$

3.3 Approximation of the Learning Curve Cost Term

Ideally, this exact but nonlinear form of the budget constraint would be used in the optimization. However, the size of the problem makes this approach impractical. Our goal is to approximate the learning curve terms in these constraints using a piecewise linear function. For cumulative average theory learning curves this process is simple. Unit theory learning curves require more work since they are nonseparable with respect to the decision variables of the problem. In this section we reformulate the learning curve terms in a way that will make the them separable, and then correct for any error that arises as the result of this change in form.

For the case of cumulative average theory learning curves, we write the cost of producing a lot of system i in year j as

$$\hat{c}_{ij}x_{ij} = (A_i y_{ij}^{-b_i})y_{ij} - (A_i y_{ij-1}^{-b_i})y_{ij-1}, \quad (3)$$

where \hat{c}_{ij} is the average unit cost of the items produced in the lot. Note that the right hand side of this expression is separable using the cumulative total quantity of system i produced through year j , y_{ij} . Furthermore, each term on the right hand side is concave, and easy to approximate in a piecewise linear manner.

For unit theory learning curves, we know that $\hat{c}_{ij}x_{ij} = A_i Q_{ij}^{-b_i} x_{ij}$ where Q_{ij} is computed using (1). In order to achieve the desired separability we consider the following approximation:

$$\hat{c}_{ij}x_{ij} \approx A_i \tilde{Q}_{ij}^{-b_i} y_{ij} - A_i \tilde{Q}_{ij-1}^{-b_i} y_{ij-1} \text{ where } \tilde{Q}_{ij} = \left[\frac{(y_{ij})(1-b_i)}{(y_{ij}+0.5)^{1-b_i} (0.5)^{1-b_i}} \right]^{\frac{1}{b_i}},$$

representing the lot midpoint of a lot of size y_{ij} . Note that the terms on the right hand side of this expression are also separable with respect to the cumulative quantities produced.

We now examine the error in using this approximation. Our purpose is to make an adjustment to the approximation so that it will be equivalent to (3). Then, using y_{ij} as the decision variable, we will have separable terms in the budget constraints that can be approximated in a piecewise linear manner.

Letting $c_{ij} = A_i \tilde{Q}_{ij}^{1-b_i}$, compute the error introduced, D_{ij} , as follows:

$$D_{ij} = (c_{ij}y_{ij} - c_{ij-1}y_{ij-1}) - \hat{c}_{ij}x_{ij}.$$

We note that this error function is also nonlinear and nonseparable. No apparent benefit has been gained by this approximation. However, when we evaluate the error function over the range of the parameters experienced in the problem under consideration we observe that the magnitude of D_{ij} is very small in comparison to the corresponding learning curve cost term in the budget constraint. That is,

$$D_{ij} \ll \hat{c}_{ij}x_{ij}.$$

Furthermore, when the error term is expressed as a proportion of the approximation, M_{ij} , we find that its value is very stable for given values of A_i and b_i for all but the first year of production, over relevant ranges of x_{ij} for all the systems that we have seen. That is,

$$M_{ij} = \frac{D_{ij}}{c_{ij}Y_{ij} - c_{ij-1}Y_{ij-1}} \approx \text{constant.} \quad (4)$$

An exception exists in the first year of production. In that year, $D_{ij} = M_{ij} = 0$ since the approximation is exactly correct.

Figures 2a and 2b give examples of these observations for two systems whose costs are described by learning curves. Figure 2a shows an example of a learning curve system that has a fairly severe curvature. Note that even in this extreme case, the worst case error is small. Figure 2b represents a more typical case where M_{ij} is almost constant over the years. The consistency of these values is surprising, since the yearly quantities vary from 1,200 to 20,000.

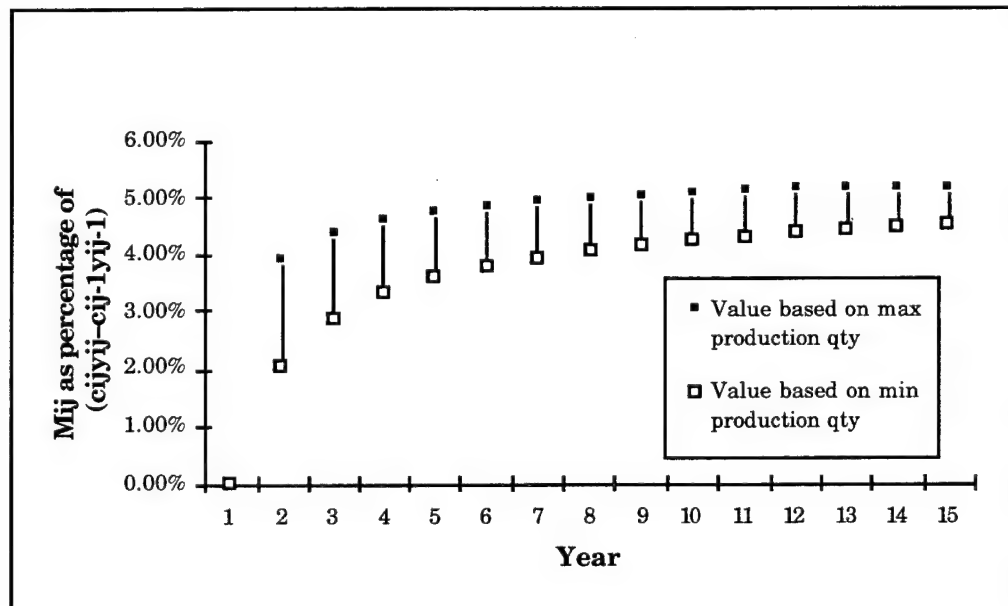


Figure 2a. Example of M_{ij} Values ($A_i=3.28$, $b_i=.258$)

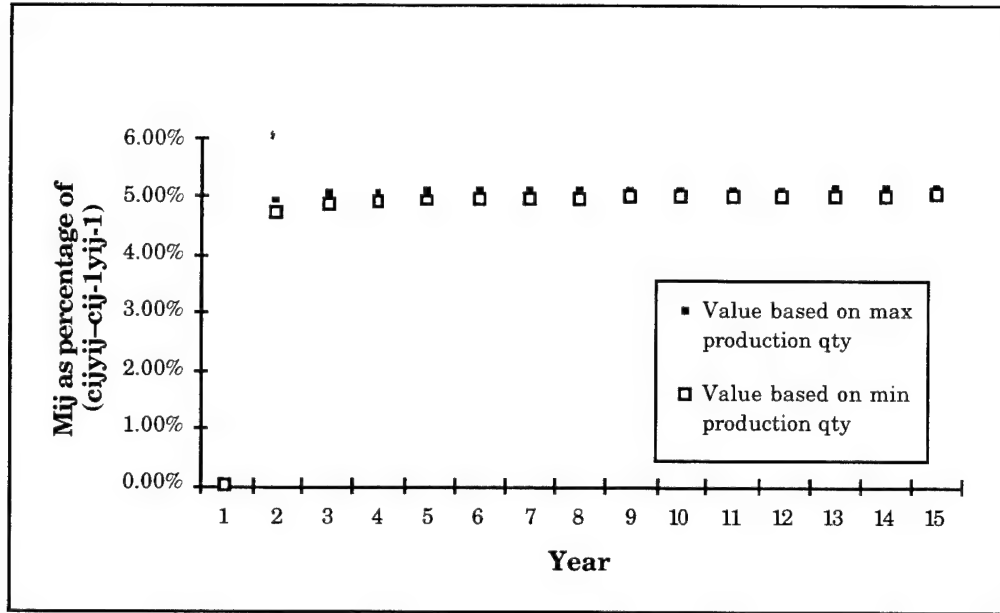


Figure 2b. Example of M_{ij} Values ($A_i=.1796$, $b_i=.1901$)

We estimate the value of each correction proportion, M_{ij} , by computing the upper and lower feasible cumulative production quantities, y_{ij} , that arise by summing the corresponding maximum feasible values of the production quantities, x_{ik} , $k=1, \dots, j$, and summing the minimum feasible values of those same quantities. So we have

$$y_{ij} \in [y_{ij\min}, y_{ij\max}], \text{ where}$$

$$y_{ij\min} = \sum_{k=1}^j x_{ik\min} \text{ and } y_{ij\max} = \sum_{k=1}^j x_{ik\max}.$$

Using (4), and substituting $y_{ij\min}$ and $y_{ij-1\min}$ for y_{ij} and y_{ij-1} , respectively, we obtain $M_{ij\min}$. Following the same procedure, but this time using the maximum values of y_{ij} and y_{ij-1} , we get $M_{ij\max}$. Empirically, we have found that the arithmetic average of these two quantities gives a good value for M_{ij} . That is

$$M_{ij} = \frac{M_{ij\max} + M_{ij\min}}{2}. \quad (5)$$

Using this method, we note that the error for each production year by applying this correction would be less than 1 percent for the system shown in Figure 2a, and within 2 percent for all the candidate systems examined in the VAA Study.

We can now rewrite the learning curve cost terms using (5) in the following manner. For each year j , we have:

$$\sum_{i=1}^m \hat{c}_{ij} x_{ij} \leq B_j \Leftrightarrow \sum_{i=1}^m (1 - M_{ij}) (c_{ij} y_{ij} - c_{ij-1} y_{ij-1}) \leq B_j. \quad (6)$$

Note that for the cumulative average learning curve model,

$M_{ij} = 0$, and $c_{ij} = A_i y_{ij}^{-b_i}$. By using this approximation and by using y_{ij} as the decision variable, these learning curve terms become separable, and although they are still nonlinear, they can be dealt with using a piecewise linear approximation.

3.4 Piecewise Linear Approximation of the Learning Curve Term

Now that the learning curve cost terms have been made separable, we can approximate them as a sequence of linear pieces. The technique used here is standard and is described by Bradley, Hax, and Magnanti [1]. Recall that each cost term is of the form

$$f(y_{ij}) = \begin{cases} A_i \tilde{Q}_{ij}^{-b_i} y_{ij}, & \text{for unit theory, and} \\ A_i y_{ij}^{-b_i} y_{ij} = A_i y_{ij}^{1-b_i}, & \text{for cumulative average theory.} \end{cases}$$

This function is graphed for one of the systems included in the Value Added Analysis, and is shown in Figure 3. This system is the same one that was discussed with regard to Figure 2a, and the unit theory curve is shown.

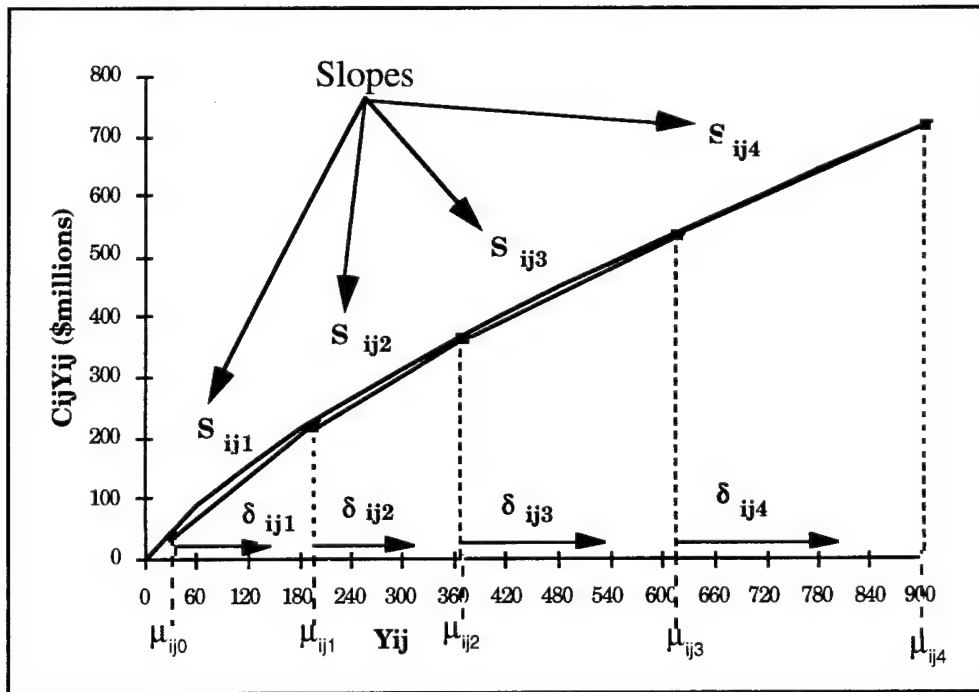


Figure 3. Piecewise Approximation of Cost Curve

It is easy to show that this function is concave. It is also very smooth. We have found that, over the range of possible values of y_{ij} , that the number of segments needed to approximate this curve varies depending on the parameters of the learning curve and the range of feasible values for y_{ij} . The relevant range of y_{ij} is defined as follows. The minimum value of y_{ij} is the minimum production quantity in the first year of production. Define this quantity as μ_{ij0} . The maximum value of y_{ij} is the sum of the maximum production quantities over all the years of production. Define this maximum quantity as μ_{ijp} , where p = the number of segments in the approximation.

To find the end points of the line segments that approximate $f(y_{ij})$, we find $\mu_{ij1}, \mu_{ij2}, \dots, \mu_{ijp-1}$, values of y_{ij} , such that the following relationship of the derivatives holds.

$$f'(\mu_{ij1}) - f'(\mu_{ij0}) = f'(\mu_{ij2}) - f'(\mu_{ij1}) = \dots = f'(\mu_{ijp}) - f'(\mu_{ijp-1}).$$

Next, we define the following variables.

δ_{ijk} = the amount greater than μ_{ijk} , where

$$0 \leq \delta_{ijk} \leq \mu_{ijk} - \mu_{ijk-1}; k = 1, \dots, p;$$

so that

$$y_{ij} = \mu_{ij0} + \delta_{ij1} + \dots + \delta_{ijp}. \quad (7)$$

$$w_{ijk} = \begin{cases} 1, & \text{if } \delta_{ijk} = \mu_{ijk} - \mu_{ijk-1}, \\ 0, & \text{if } \delta_{ijk} < \mu_{ijk} - \mu_{ijk-1}. \end{cases} \quad (8)$$

The purpose of these binary variables, w_{ijk} , is to ensure that δ_{ijk} will never be positive unless δ_{ijk-1} is at its maximum, for $2 \leq k \leq p$. These conditions are enforced through the use of the following constraints.

$$\begin{aligned}
(\mu_{ij1} - \mu_{ij0})w_{ij1} &\leq \delta_{ij1} \leq \mu_{ij1} - \mu_{ij0}, \\
(\mu_{ij2} - \mu_{ij1})w_{ij2} &\leq \delta_{ij2} \leq (\mu_{ij2} - \mu_{ij1})w_{ij1}, \\
&\bullet \\
&\bullet \\
&\bullet \\
&\bullet \\
(\mu_{ijp-1} - \mu_{ijp-2})w_{ijp-1} &\leq \delta_{ijp-1} \leq (\mu_{ijp-1} - \mu_{ijp-2})w_{ijp-2}, \\
0 \leq \delta_{ijp} &\leq (\mu_{ijp} - \mu_{ijp-1})w_{ijp-1}.
\end{aligned} \tag{9}$$

Notice that the binary variables, w_{ijk} , act as switches for the δ_{ijk} variables such that, when δ_{ij1} is pushed to its maximum allowable value, $\mu_{ij1} - \mu_{ij0}$, w_{ij1} is toggled from a value of 0 to a value of 1. Thus, δ_{ij2} , previously constrained to be zero, is allowed to grow toward its own maximum value. Now, calculate the slopes of each segment. Let

$$S_{ijk} = \frac{f(\mu_{ijk}) - f(\mu_{ijk-1})}{\mu_{ijk} - \mu_{ijk-1}}. \tag{10}$$

Then we can approximate each cost term as

$$c_{ij}y_{ij} = f(y_{ij}) \approx f(\mu_{ij0}) + \sum_{k=1}^p S_{ijk} \delta_{ijk}, \tag{11}$$

with the additional constraints that

$$y_{ij} = \mu_{ij0} + \sum_{k=1}^p \delta_{ijk}, \quad \forall i, j. \tag{12}$$

The approximation in (6) is then written as

$$\sum_{i=1}^m (1 - M_{ij}) \left(f(\mu_{ij0}) + \sum_{k=1}^p S_{ijk} \delta_{ijk} - f(\mu_{ij-10}) - \sum_{k=1}^p S_{ij-1k} \delta_{ij-1k} \right) \leq B_j, \tag{13}$$

with constraints (9) and (12), and using the convention that μ_{i00} and δ_{i00} are defined to be zero.

This approximation, then, introduces $2p-1$ variables, of which $p-1$ are binary, and $2p$ constraints to the formulation of the problem for each y_{ij} variable. The dimensionality of the problem is thus greatly increased.

4. IMPLEMENTATION AND PERFORMANCE

This optimization model was implemented on an IBM RS 6000, model 590 work station using the IBM Optimization Software Library (OSL). See IBM Corporation [7]. We elected to write a front-end application program in FORTRAN that reads the data, processes the data into the appropriate data structures for the optimizer, calls the optimization subroutines, and then prints the results.

Frequently, the speed of solution of a mixed integer program can be improved by exploiting relationships between the integer variables and introducing additional constraints in the formulation. A good discussion of the techniques used to identify the additional constraints is given by Johnson and Nemhauser [6]. Although the internal OSL preprocessor failed to find any adjustments to the formulation that would improve its performance, a close examination of the structure of the formulation yielded several performance enhancing adjustments.

The first set of constraints introduced was based on the relationship between the binary variables that are used to implement the piecewise linear approximation of cost curve. Recall that we include a cost curve for each system i for each year j of production, introducing several binary variables, w_{ijk} , into the formulation, the number depending on the number of linear pieces employed in the approximation. Since the cost curves are based on the cumulative quantity of system i procured over the production years, y_{ij} , we know that $y_{ij} \geq y_{ij-1}$. Thus, using (12), we know that $\delta_{ijk} \geq \delta_{ij-1k}$ for all $j > 1$. Therefore, we can impose the following constraints on the w_{ijk} binary variables.

$$w_{ijk} \geq w_{ij-1k}; \text{ for all } j > 1, i = 1, \dots, m, k = 1, \dots, p.$$

When these constraints are imposed, branches become much more powerful in the sense that setting one variable to the value 0 potentially sets many others as well.

Similarly, we see immediately that all the w_{ijk} variables will be 0 unless the u_i variables are set at 1. So, another set of constraints that is implicit in the formulation but whose explicit inclusion improves the performance of the algorithm are expressed as follows:

$$u_i \geq w_{ijk}, \text{ for all } j \geq 1, \text{ for all } i = 1, \dots, m, k = 1, \dots, p.$$

These constraints ensure that all w_{ijk} variables are set to 0 if u_i is 0, making branches on the u_i variables very powerful.

As the result of the additional 500 constraints, the performance of the model was much improved. Ultimately, 45 systems were analyzed, of which 22 had learning curve costs. The mixed integer program had about 4,000 rows with 3,000 variables, of which about 800 were binary integers, and 5,500 nonzero elements.

The run time for this improved formulation was reduced to between 2 and 45 minutes of CPU time, with an average of 20 minutes. Without the performance-enhancing modifications, the model would not have been as responsive as was necessary to provide the required analytical support.

5. CONCLUSIONS

The methodology introduced in this paper seems to do a good job of incorporating the learning curve effects on costing into the budget constraints of the Value Added Analysis acquisition strategy optimization. The introduction of this feature greatly increases the computational overhead associated with solving problems of this nature. As a result, implementation of this enhancement to acquisition strategy models requires significantly increased computing resources to obtain a solution.

This methodology is an approximation that has yielded results in which the expended program dollars, calculated using the nonlinear cost function and the optimized quantities, were within two percent of the nominal value. Considering the approximate nature of costing systems that will only be procured in the far distant future, two percent is adequate.

The use of this methodology has been shown to improve the quality of the optimization for the purpose of acquisition strategy by maintaining consistency between the costs used in the model and those used by the programming and budgeting staff. In this era of tightly constrained budgets for procurement, accurate cost analysis is essential to obtain the most from limited funds. This methodology has enhanced analytical efforts that help accomplish this task. This optimization model was successfully used to assist the Army Staff in evaluating the various alternative weapon systems considered for procurement. As such, this optimization model has provided a new dimension to the PPBES process for the Department of the Army Staff. The Staff now has available in a single model the capability to pull together data, policy, and guidance quickly and accurately in order to develop a balanced Army program.

REFERENCES

- [1] Bradley, Stephen R., Arnolddo C. Hax, and Thomas L. Magnanti, *Applied Mathematical Programming*, Addison-Wesley, Reading, MA, 1977.
- [2] Brown, Gerald G., Robert D. Clemence, William R. Teufert, and R. Kevin Wood, "An Optimization Model for Modernizing the Army's Helicopter Fleet," *Interfaces*, 21, July - August 1991, 39-52.
- [3] Coblentz, Linda A., "Linear Programming Analysis Tools for Acquisition Strategies: The Phoenix and Force Modernization Analyzer (FOMOA) Models," *Proceedings from the Symposium on Upgrades Versus Modernization*, Technical Proceedings AC/243 (Panel 7) TP/5, September 9, 1991, 4.1-37.

- [4] Department of Defense Manual Number 5000.2-M, Part 17, February 1991, 17-1-25.
- [5] Dutton, John M., Annie Thomas, and John E. Butler, "The History of Progress Functions as a Management Technology," *Business History Review*, 58, 2 (Summer 1984), 204-33.
- [6] Johnson, Ellis L. and George L. Nemhauser, Recent Developments and Future Directions in Mathematical Programming, Research Paper RC 17234 (#76333), IBM Research Division, September 30, 1991.
- [7] IBM Corporation, *Optimization Subroutine Library, Guide and Reference*, Release 2, SC23-0519-02, 1991.
- [8] Kanton, Paul B. and Willard I. Zangwill, "Theoretical Foundation for a Learning Rate Budget," *Management Science*, Vol. 37, 3, (March 1991), 315-330.
- [9] Koury, Robert R. and Andrew G. Loerch, Army Program Value Added Analysis 94-99 (VAA II) Study, CAA-SR-92-10, US Army Concepts Analysis Agency, Bethesda, MD, 1992.
- [10] Levy, Joel, Resource Constrained Procurement Objectives for Munitions (RECPOM-85) Study, CAA-SR-86-23, US Army Concepts Analysis Agency, Bethesda, MD, 1986.
- [11] Pearlstein, Steven, "Cuts in Defense Budget Create New Inefficiencies," *Washington Post*, 339 (November 9, 1991), A1.
- [12] Schwabauer, Robert and Edward Nedimala, "Spreadsheets, Optimization, and FOMOA," *Proceedings of the 29th US Army Operations Research Symposium*, October 9, 1990, Vol. II.
- [13] Stump, Evin Jr., "Composite Learning Curves for Fast Estimating," *Journal of Cost Analysis*, Vol 5, 1, Summer 1987, 59-69.
- [14] U.S. Army Regulation 1-1, Planning, Programming, Budgeting, and Execution System, Washington D.C., February 1990.
- [15] Wright, T.P., "Factors Affecting the Cost of Airplanes," *J. Aeronautical Science*, 3 (1936), 122-28.
- [16] Yelle, Louis E., "The Learning Curve: Historical Review and Comprehensive Survey," *Decision Sciences*, 10 (1979), 302-328.